Algebraic properties of connectors in weighted architectures

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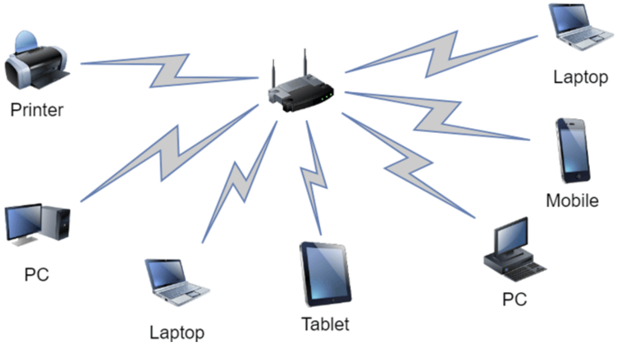
Motivation (1)

- Complex systems usually consist of several components.



But how are they connected?

Motivation (2)

- Complex systems are modeled by component-based approaches.
- Component-based modeling lies in constructing systems by coordinating and composing multiple components.
- Communication is performed through their interfaces, i.e., their **ports**.
- Coordination can be described by **interactions**, i.e., sets of ports.

Motivation (3)

- Communication of components can be formalized by **architectures** or **connectors**.
- **Connectors** express efficiently synchronization constraints between the components' activities.
- **Connectors** achieve to encode versatile coordination schemes, e.g., Rendezvous or Broadcast.
- **Architectures** encode both the coordination schemes and the structure of the components.

Related work

<u>**Question:**</u>How can we model connectors in component-based systems?

<u>**Answer:**</u> The Algebra of Connectors can describe the qualitative features of the components' communication.

S. Bliudze, J. Sifakis, The algebra of connectors - Structuring interaction in BIP, *IEEE Trans. Computers* 57(10)(2008) 1315–1330. doi:10.1109/TC.2008.26

Problem to be solved

What about the quantitative aspects of the components' communication?

- Rigorous design should incorporate the quantitative aspects.
- For instance, the total cost or the maximum probability of implementing specific connections.

In this Thesis, we address the formal modeling of connectors in the weighted setup.

- We propose and study the weighted Algebra of Connectors.

Summary

1. Preliminary notions.
2. Weighted BIP Component Framework (wBIP).
3. Weighted Algebra of Interactions.
4. Weighted Algebra of Connectors.
5. Weighted subalgebras.
6. Open problems.

Preliminary notions

In this Thesis, the weights range over a totally ordered commutative and idempotent semiring $(\boldsymbol{K}, \oplus, \otimes, \hat{\mathbf{0}}, \hat{\mathbf{1}})$.

- The semiring K is called **idempotent** if $(K, \oplus, \hat{0})$ is an idempotent monoid, i.e., $k \oplus k = k$ for every $k \in K$.
- Let P be a non-empty set.
- A **formal series** (or simply **series**) over P and K is a mapping $s : P \to K$.
- The support of s is the set $\mathrm{supp}(s) = \{p \in P \mid s(p) \neq \hat{0}\}$.
- A series with finite support is called a **polynomial**.
- We denote by $K\langle P \rangle$ the class of all polynomials over P and K.

Preliminary notions

Definition

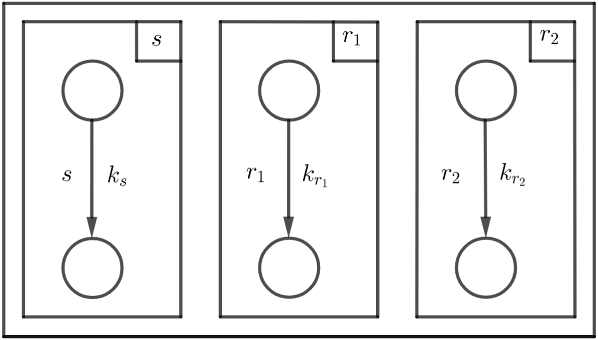
Let P be a finite non-empty set of ports. Then an **interaction** a *is a non-empty set of ports over P, i.e.,* $a \in 2^P \setminus \{\emptyset\}$.

- We let $\Gamma(P) = 2^{2^P}$.
- For every natural number $n \geq 1$ we denote by $[n]$ the set $\{1, \ldots, n\}$.

Weighted BIP component framework (wBIP)

First layer of wBIP

1. The **weighted Behavior layer** is modeled by **weighted labeled transition systems**.



Weighted BIP component framework (wBIP)

Second and third layer of wBIP

2. The **weighted Interaction layer**, is modeled by the components' composition and is parameterized by the set of the allowed **interactions** associated with weights over K.

3. The **weighted Priorities layer**, restricts the several interactions that can be enabled at the same time, and the most **preferable interaction** is chosen by its corresponding weight.

Weighted algebra of interactions

Syntax

Definition

- *Consider a weighted system of n components* $\{wB_i\}_{i=1}^{n}$ *for* $i \in [n]$ *and let* $P = \cup_{i \in [n]} P_i$.
- *The* **syntax** *of* $wAI(P)$ *over* P *and* K *is defined by*

$$z ::= 0 \mid 1 \mid p \mid z \oplus z \mid z \otimes z \mid (z)$$

where $p \in P$, $0, 1 \notin P$, "$\oplus$" *is the* **weighted union operator** *and* "$\otimes$" *the* **weighted synchronization operator**.

Weighted algebra of interactions

Semantics

Definition

Let z be a $wAI(P)$ component over P and K. The **semantics** *of z is a polynomial $\|z\| \in K\langle\Gamma(P)\rangle$. For every $\gamma \in \Gamma(P)$, the value $\|z\|(\gamma)$ is defined inductively on z as follows:*

- $\|0\|(\gamma) = \hat{0}$,
- $\|1\|(\gamma) = \begin{cases} \hat{1} & \text{if } \emptyset \in \gamma \\ \hat{0} & \text{otherwise} \end{cases}$,
- $\|p\|(\gamma) = \begin{cases} k_p & \text{if } \exists\, a \in \gamma \text{ such that } p \in a \\ \hat{0} & \text{otherwise} \end{cases}$,
- $\|z_1 \oplus z_2\|(\gamma) = \bigoplus_{a\in\gamma} \left(\|z_1\|(\{a\}) \oplus \|z_2\|(\{a\})\right)$,
- $\|z_1 \otimes z_2\|(\gamma) = \bigoplus_{a\in\gamma} \left(\bigoplus_{a=a_1\cup a_2} \left(\|z_1\|(\{a_1\}) \otimes \|z_2\|(\{a_2\})\right)\right)$,
- $\|(z)\|(\gamma) = \|z\|(\gamma)$.

Equivalence of $wAI(P)$ components

- Two components $z_1, z_2 \in wAI(P)$ are **equivalent** and we write $z_1 \equiv z_2$, if $\|z_1\|(\gamma) = \|z_2\|(\gamma)$ for every $\gamma \in \Gamma(P)$.
- The relation "$\equiv$" is an equivalence relation.
- The structure $(wAI(P)/\equiv, \oplus, \otimes, \bar{0}, \bar{1})$ is a commutative and idempotent semiring.

Example (1)

- Consider two $wAI(P)$ components with ports s, r, respectively.
- We consider the $wAI(P)$ component $s \otimes (1 \oplus r)$.
- Let $\gamma = \{\{s\}, \{s, r\}\}$.
- For $\boldsymbol{a = \{s\}} \in \gamma$ we have:

$\|s \otimes (1 \oplus r)\| (\{a\})$			
$a = a_1 \cup a_2$	$\|s\| (\{a_1\})$	$\|1 \oplus r\| (\{a_2\})$	$\otimes$
$a_1 = \emptyset, a_2 = \{s\}$	$\hat{0}$	$\hat{0} \oplus \hat{0}$	$\hat{0}$
$a_1 = \{s\}, a_2 = \emptyset$	k_s	$\hat{1} \oplus \hat{0}$	k_s
$a_1 = \{s\}, a_2 = \{s\}$	k_s	$\hat{0} \oplus \hat{0}$	$\hat{0}$
$\oplus$			k_s

Example (2)

- For $\boldsymbol{a = \{s, r\}} \in \gamma$ where $\gamma = \{\{s\}, \{s, r\}\}$ we have:

$\|s \otimes (1 \oplus r)\| (\{a\})$			
$a = a_1 \cup a_2$	$\|s\| (\{a_1\})$	$\|1 \oplus r\| (\{a_2\})$	$\otimes$
$a_1 = \emptyset, a_2 = \{s, r\}$	$\hat{0}$	$\hat{0} \oplus k_r$	$\hat{0}$
$a_1 = \{s, r\}, a_2 = \emptyset$	k_s	$\hat{1} \oplus \hat{0}$	k_s
$a_1 = \{s\}, a_2 = \{r\}$	k_s	$\hat{0} \oplus k_r$	$k_s \otimes k_r$
$a_1 = \{r\}, a_2 = \{s\}$	$\hat{0}$	$\hat{0} \oplus \hat{0}$	$\hat{0}$
$a_1 = \{s\}, a_2 = \{s, r\}$	k_s	$\hat{0} \oplus k_r$	$k_s \otimes k_r$
$a_1 = \{s, r\}, a_2 = \{s\}$	k_s	$\hat{0} \oplus \hat{0}$	$\hat{0}$
$a_1 = \{r\}, a_2 = \{s, r\}$	$\hat{0}$	$\hat{0} \oplus k_r$	$\hat{0}$
$a_1 = \{s, r\}, a_2 = \{r\}$	k_s	$\hat{0} \oplus k_r$	$k_s \otimes k_r$
$a_1 = \{s, r\}, a_2 = \{s, r\}$	k_s	$\hat{0} \oplus k_r$	$k_s \otimes k_r$
$\oplus$			$k_s \oplus (k_s \otimes k_r)$

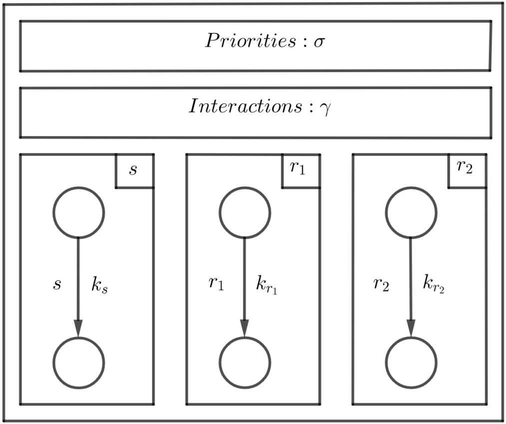
Example (3)

Applying the semantics of the $wAI(P)$ we get

$$\begin{aligned}
&\left\|s \otimes (1 \oplus r)\right\|(\gamma) \\
=&\bigoplus_{a \in \gamma} \Big(\bigoplus_{a = a_1 \cup a_2} \big(\|s\| (\{a_1\}) \otimes \|1 \oplus r\| (\{a_2\}) \big) \Big) \\
=&\bigoplus_{a \in \gamma} \left(\bigoplus_{a = a_1 \cup a_2} \left(\|s\| (\{a_1\}) \otimes \Big(\bigoplus_{a' \in \{a_2\}} \big(\|1\| (\{a'\}) \oplus \|r\| (\{a'\}) \big) \Big) \right) \right) \\
=&\bigoplus_{a \in \gamma} \left(\bigoplus_{a = a_1 \cup a_2} \left(\|s\| (\{a_1\}) \otimes \Big(\|1\| (\{a_2\}) \oplus \|r\| (\{a_2\}) \Big) \right) \right) \\
=&k_s \oplus \big(k_s \oplus (k_s \otimes k_r)\big) \\
=&k_s \oplus (k_s \otimes k_r).
\end{aligned}$$

Weighted coordination schemes

- Consider a sender wS and two receivers wR_1, wR_2.
- The ports are s and r_1, r_2 with weights k_s, k_{r_1}, k_{r_2}, respectively.



Application on weighted Rendezvous (1)

- Strong synchronization between the participating ports, described by the $wAI(P)$ component

$$\boldsymbol{s} \otimes \boldsymbol{r_1} \otimes \boldsymbol{r_2}.$$

- Let $\gamma = \{\{s, r_1, r_2\}\}$.

$$\begin{aligned}
&\|s \otimes r_1 \otimes r_2\| (\gamma) \\
=&\bigoplus_{a\in\gamma} \Big(\bigoplus_{a=a_1\cup a_2} \big(\|s\| (\{a_1\}) \otimes \|r_1 \otimes r_2\| (\{a_2\})\big)\Big) \\
=&\bigoplus_{a\in\gamma} \left(\bigoplus_{a=a_1\cup a_2} \left(\|s\| (\{a_1\}) \otimes \left(\bigoplus_{a'\in\{a_2\}} \left(\bigoplus_{a'=a_{2,1}\cup a_{2,2}} \big(\|r_1\| (\{a_{2,1}\}) \otimes \|r_2\| (\{a_{2,2}\})\big)\right)\right)\right)\right) \\
=&\bigoplus_{a\in\gamma} \left(\bigoplus_{a=a_1\cup a_2} \left(\|s\| (\{a_1\}) \otimes \left(\bigoplus_{a_2=a_{2,1}\cup a_{2,2}} \big(\|r_1\| (\{a_{2,1}\}) \otimes \|r_2\| (\{a_{2,2}\})\big)\right)\right)\right) \\
=&k_s \otimes k_{r_1} \otimes k_{r_2}.
\end{aligned}$$

Application on weighted Rendezvous (2)

- Depending on the semiring being used, the semantics of the weighted component has a different meaning.
- For the fuzzy semiring $F = ([0,1], \max, \min, 0, 1)$ we get the maximum of the minimum weights associated with each port in the system:

$$
\begin{aligned}
&\|s \otimes r_1 \otimes r_2\|(\gamma) \\
=&\max_{a\in\gamma}\left(\max_{a=a_1\cup a_2}\left(\min\left(\|s\|(\{a_1\}), \max_{a_2=a_{2,1}\cup a_{2,2}}(\min(\|r_1\|(\{a_{2,1}\}), \|r_2\|(\{a_{2,2}\})))\right)\right)\right) \\
=&\max_{a\in\gamma}\left(\max_{a=a_1\cup a_2}\left(\max_{a_2=a_{2,1}\cup a_{2,2}}\left(\min\left(\|s\|(\{a_1\}), \min(\|r_1\|(\{a_{2,1}\}), \|r_2\|(\{a_{2,2}\}))\right)\right)\right)\right) \\
=&\max_{a\in\gamma}\left(\max_{a=a_1\cup a_{2,1}\cup a_{2,2}}\left(\min\left(\|s\|(\{a_1\}), \|r_1\|(\{a_{2,1}\}), \|r_2\|(\{a_{2,2}\})\right)\right)\right).
\end{aligned}
$$

Application on weighted Broadcast (1)

- Executes all interactions involving wS and any subset wR_i, for $i = 1, 2$, possibly an empty one, described by the $wAI(P)$ component

$$\boldsymbol{s} \otimes (\mathbf{1} \oplus \boldsymbol{r_1}) \otimes (\mathbf{1} \oplus \boldsymbol{r_2}).$$

- Let $\gamma = \{\{s\}, \{s, r_1\}, \{s, r_2\}, \{s, r_1, r_2\}\}$.

$$\left\|s \otimes (1 \oplus r_1) \otimes (1 \oplus r_2)\right\|(\gamma)$$

$$= \bigoplus_{a\in\gamma} \Big(\bigoplus_{a=a_1\cup a_2} \left(\|s\| (\{a_1\}) \otimes \|(1 \oplus r_1) \otimes (1 \oplus r_2)\| (\{a_2\})\right)\Big)$$

$$= \bigoplus_{a\in\gamma} \left(\bigoplus_{a=a_1\cup a_2} \left(\|s\| (\{a_1\}) \otimes \left(\bigoplus_{a'\in\{a_2\}} \left(\bigoplus_{a'=a_{2,1}\cup a_{2,2}} \left(\|1 \oplus r_1\| (\{a_{2,1}\})\otimes \right.\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\left. \|1 \oplus r_2\| (\{a_{2,2}\})\right)\right)\right)\right)\right)$$

$$= \bigoplus_{a\in\gamma} \left(\bigoplus_{a=a_1\cup a_2} \left(\|s\| (\{a_1\}) \otimes \left(\bigoplus_{a_2=a_{2,1}\cup a_{2,2}} \left(\|1 \oplus r_1\| (\{a_{2,1}\}) \otimes \|1 \oplus r_2\| (\{a_{2,2}\})\right)\right)\right)\right)$$

Application on weighted Broadcast (2)

$$=\bigoplus_{a\in\gamma}\left(\bigoplus_{a=a_1\cup a_2}\left(\|s\|(\{a_1\})\otimes\left(\bigoplus_{a_2=a_{2,1}\cup a_{2,2}}\left(\bigoplus_{a''\in\{a_{2,1}\}}(\|1\|(\{a_{2,1}\})\oplus\|r_1\|(\{a_{2,1}\}))\otimes\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\bigoplus_{a'''\in\{a_{2,2}\}}(\|1\|(\{a_{2,2}\})\oplus\|r_2\|(\{a_{2,2}\}))\right)\right)\right)\right)$$

$$=\bigoplus_{a\in\gamma}\left(\bigoplus_{a=a_1\cup a_2}\left(\|s\|(\{a_1\})\otimes\left(\bigoplus_{a_2=a_{2,1}\cup a_{2,2}}\left((\|1\|(\{a_{2,1}\})\oplus\|r_1\|(\{a_{2,1}\}))\otimes\right.\right.\right.\right.$$

$$\left.\left.\left.\left.(\|1\|(\{a_{2,2}\})\oplus\|r_2\|(\{a_{2,2}\}))\right)\right)\right)\right)$$

$$=k_s\oplus(k_s\oplus(k_s\otimes k_{r_1}))\oplus(k_s\oplus(k_s\otimes k_{r_2}))\oplus(k_s\oplus(k_s\otimes k_{r_1})\oplus(k_s\otimes k_{r_2})\oplus$$
$$(k_s\otimes k_{r_1}\otimes k_{r_2}))$$

$$=k_s\oplus(k_s\otimes k_{r_1})\oplus(k_s\otimes k_{r_2})\oplus(k_s\otimes k_{r_1}\otimes k_{r_2}).$$

Application on weighted Broadcast (3)

- Consider for instance $\mathbb{R}_{\max} = (\mathbb{R}_+ \cup \{-\infty\}, \max, +, -\infty, 0)$.
- The resulting value represents the total cost of the maximum weights on γ:

$$\left\| s \otimes (1 \oplus r_1) \otimes (1 \oplus r_2) \right\| (\gamma)$$

$$= \max_{a \in \gamma} \Bigg(\max_{a = a_1 \cup a_2} \Bigg(\|s\| (\{a_1\}) + \max_{a_2 = a_{2,1} \cup a_{2,2}} \Big(\max\big(\|1\| (\{a_{2,1}\}), \|r_1\| (\{a_{2,1}\}) \big) +$$

$$\max\big(\|1\| (\{a_{2,2}\}), \|r_2\| (\{a_{2,2}\}) \big) \Big) \Bigg) \Bigg).$$

Weighted Atomic Broadcast and Causality Chain

- **Weighted Atomic Broadcast**: A message is either received by all receivers wR_i, for $i = 1, 2$, or by none, described by the $wAI(P)$ component

$$\boldsymbol{s} \otimes (\mathbf{1} \oplus \boldsymbol{r_1} \otimes \boldsymbol{r_2}).$$

- **Weighted Causality Chain**: If a message is received by receiver wR_i for $i = 1, 2$, then it has to be received by all receivers wR_j, for $j < i$, described by the $wAI(P)$ component

$$\boldsymbol{s} \otimes (\mathbf{1} \oplus \boldsymbol{r_1} \otimes (\mathbf{1} \oplus \boldsymbol{r_2})).$$

Weighted algebra of connectors

Syntax

Definition

Let P be a set of ports, such that $0, 1 \notin P$. The **syntax** *of $wAC(P)$ over P and K is defined by*

$$\sigma ::= [0] \mid [1] \mid [p] \mid [\zeta] \quad (synchron)$$

$$\tau ::= [0]' \mid [1]' \mid [p]' \mid [\zeta]' \quad (trigger)$$

$$\zeta ::= \sigma \mid \tau \mid \zeta \oplus \zeta \mid \zeta \otimes \zeta$$

where $p \in P$, " $\oplus$ " denotes the **weighted union operator**, *" $\otimes$ " denotes the* **weighted fusion operator**, *and " $[\cdot]$ ", " $[\cdot]'$ " are the synchron and trigger* **typing operators**.

- **Triggers** denoted by "$[\cdot]'$" are responsible for initiating an interaction.
- **Synchrons** denoted by "$[\cdot]$" need synchronization with other ports.

Weighted algebra of connectors

Notations

- We write $[\zeta]^{\alpha}$ for $\alpha \in \{0,1\}$ to denote a typed weighted connector,
  - when $\alpha = 0$ represents a synchron element
  - when $\alpha = 1$ it is a trigger element.
- For simplicity, brackets are omitted from 0, 1, and ports $p \in P$.
- $\zeta = [\zeta_1]^{\alpha_1} \otimes \ldots \otimes [\zeta_n]^{\alpha_n} \in wAC(P)$ is called *restricted*.
- $r\text{-}wAC(P)$: the class of all restricted weighted connectors of $wAC(P)$.
- $T : r\text{-}wAC(P) \to \mathbb{N}$ returns the number of $\alpha_i \in \{0,1\}$, for $i \in [n]$ which are triggers.
- $S : r\text{-}wAC(P) \to \mathbb{N}$ returns the number of $\alpha_i \in \{0,1\}$, for $i \in [n]$ which are synchrons.

Weighted algebra of connectors

Semantics (1)

Definition

Let ζ be a $wAC(P)$ connector over P and K. The **semantics** *of ζ is a $wAI(P)$ component defined by the function $|\cdot| : wAC(P) \to wAI(P)$ as follows:*

- $\left| [p] \right| = p$, *for* $p \in P \cup \{0, 1\}$,
- $\left| [p]' \right| = p$, *for* $p \in P \cup \{0, 1\}$,
- $|[\zeta]| = |\zeta|$,
- $\left| [\zeta]' \right| = |\zeta|$,
- $|\zeta_1 \oplus \zeta_2| = |\zeta_1| \oplus |\zeta_2|$,

Weighted algebra of connectors

Semantics (2)

Definition

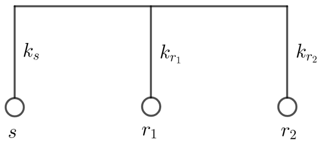
- $\big| [\zeta_1] \otimes [\zeta_2] \big| = |\zeta_1| \otimes |\zeta_2|$,
- $\big| [\zeta_1]' \otimes [\zeta_2]' \big| = \big(|\zeta_1| \otimes (1 \oplus |\zeta_2|) \big) \oplus \big(|\zeta_2| \otimes (1 \oplus |\zeta_1|) \big)$,
- $\big| [\zeta_1]^{\alpha_1} \otimes \ldots \otimes [\zeta_n]^{\alpha_n} \big| = \bigoplus_{\substack{i \in [n], \\ \alpha_i = 1}} \Big(|\zeta_i| \otimes \bigotimes_{\substack{k \neq i, \\ \alpha_k = 1}} (1 \oplus |\zeta_k|) \otimes \bigotimes_{\substack{j \in [n], \\ \alpha_j = 0}} (1 \oplus |\zeta_j|) \Big)$,

  where $T([\zeta_1]^{\alpha_1} \otimes \ldots \otimes [\zeta_n]^{\alpha_n}) \neq 0$ *and* $S([\zeta_1]^{\alpha_1} \otimes \ldots \otimes [\zeta_n]^{\alpha_n}) \neq 0$.

Equivalence on $wAC(P)$

- Two weighted connectors $\zeta_1, \zeta_2 \in wAC(P)$ are **equivalent**, and we write $\zeta_1 \equiv \zeta_2$ if $|\zeta_1| = |\zeta_2|$, i.e., whenever they return the same $wAI(P)$ components.
- This in turn implies that $\| |\zeta_1| \|(\gamma) = \| |\zeta_2| \|(\gamma)$ for every $\gamma \in \Gamma(P)$, i.e., equivalent connectors return the same weight on the same set of interactions γ.
- The relation "$\equiv$" is an equivalence relation.
- $\overline{[\zeta_1]^\alpha} \otimes \overline{[1]} = \overline{[\zeta_1]^\alpha} = \overline{[1]} \otimes \overline{[\zeta_1]^\alpha}$ for every $\zeta_1 \in wAC(P)$ and $\alpha \in \{0, 1\}$.
- We use triangles and circles in figures to represent the types triggers and synchrons, respectively.

Application on weighted Rendezvous

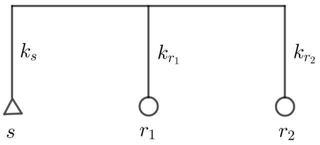


- $[\boldsymbol{s}] \otimes [\boldsymbol{r}_1] \otimes [\boldsymbol{r}_2]$

- The weighted connector requires the simultaneous participation of all ports without any typing trigger operator, and we obtain its $wAI(P)$ component as follows:

$$\begin{aligned}\big|[s] \otimes [r_1] \otimes [r_2]\big| &= |s| \otimes |r_1| \otimes |r_2| \\ &= s \otimes r_1 \otimes r_2.\end{aligned}$$

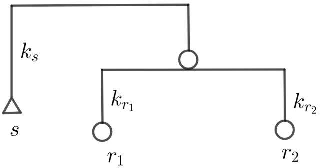
Application on weighted Broadcast



- $[\boldsymbol{s}]' \otimes [\boldsymbol{r_1}] \otimes [\boldsymbol{r_2}]$
- The weighted connector involves the trigger $[s]'$ which can initiate the interaction with the synchrons $[r_1], [r_2]$. We obtain the $wAI(P)$ component as follows:

$$\begin{aligned} \left| [s]' \otimes [r_1] \otimes [r_2] \right| &= |s| \otimes (1 \oplus |r_1|) \otimes (1 \oplus |r_2|) \\ &= s \otimes (1 \oplus r_1) \otimes (1 \oplus r_2). \end{aligned}$$

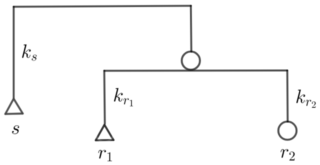
Application on weighted Atomic Broadcast



- $[\boldsymbol{s}]' \otimes \big[[\boldsymbol{r_1}] \otimes [\boldsymbol{r_2}]\big]$
- The connector consists of a trigger $[s]'$ and a synchron $\big[[r_1] \otimes [r_2]\big]$. The $wAI(P)$ component of the connector is obtained as follows:

$$\begin{aligned} \Big|[s]' \otimes \big[[r_1] \otimes [r_2]\big]\Big| &= |s| \otimes \Big(1 \oplus \big|[r_1] \otimes [r_2]\big|\Big) \\ &= |s| \otimes \big(1 \oplus |r_1| \otimes |r_2|\big) \\ &= s \otimes (1 \oplus r_1 \otimes r_2). \end{aligned}$$

Application on weighted Causality Chain



- $[\boldsymbol{s}]' \otimes \big[[\boldsymbol{r_1}]' \otimes [\boldsymbol{r_2}]\big]$

- The $wAI(P)$ component of the above connector is computed as follows:

$$\left|[s]' \otimes \big[[r_1]' \otimes [r_2]\big]\right| = |s| \otimes \Big(1 \oplus \big|[r_1]' \otimes [r_2]\big|\Big)$$
$$= |s| \otimes (1 \oplus |r_1| \otimes (1 \oplus |r_2|))$$
$$= s \otimes (1 \oplus r_1 \otimes (1 \oplus r_2)).$$

Weighted subalgebras

Weighted subalgebra of synchrons

Definition

Given a set of ports P, the **syntax** *of the weighted algebra of synchrons ($wAS(P)$ for short) over P and K is defined by:*

$$\sigma ::= [0] \mid [1] \mid [p] \mid [\zeta]$$

$$\zeta ::= \sigma \mid \zeta \oplus \zeta \mid \zeta \otimes \zeta$$

where $p \in P$, σ denotes a synchron element and $\zeta \in wAS(P)$.

Weighted subalgebras

Weighted sublgebra of triggers

Definition

Given a set of ports P, the **syntax** *of the weighted algebra of triggers ($wAT(P)$ for short) over P and K is defined by:*

$$\tau ::= [0]' \mid [1]' \mid [p]' \mid [\zeta]'$$

$$\zeta ::= \tau \mid \zeta \oplus \zeta \mid \zeta \otimes \zeta$$

where $p \in P$, τ denotes a trigger element and $\zeta \in wAT(P)$.

- $\overline{[\zeta_1]'} \otimes \overline{[0]'} = \overline{[\zeta_1]'} = \overline{[0]'} \otimes \overline{[\zeta_1]'}$ for every $\zeta_1 \in wAT(P)$.

Open problems

Future work can be oriented to:

- Relax the idempotency property of semiring K.
- Study the concept of weighted connectors over more general structures than semirings, for instance valuation monoids.
- Replacement of a $wAC(P)$ connector with a more efficient one without affecting the underlying system (weighted congruence relation).

Thank you for your attention!