Logical directed description of software architectures

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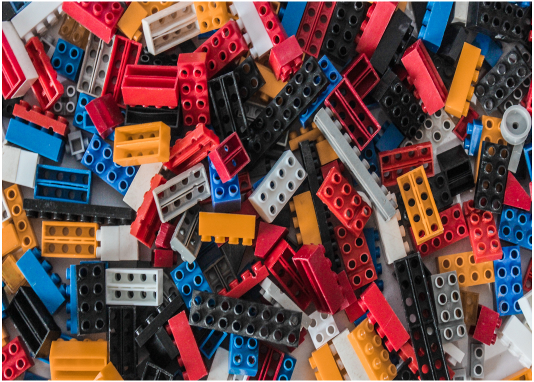
- A system consists of several components.



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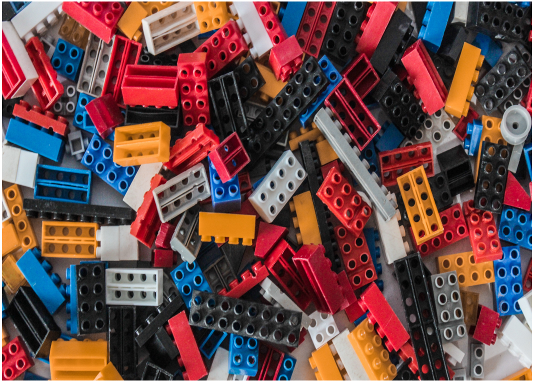
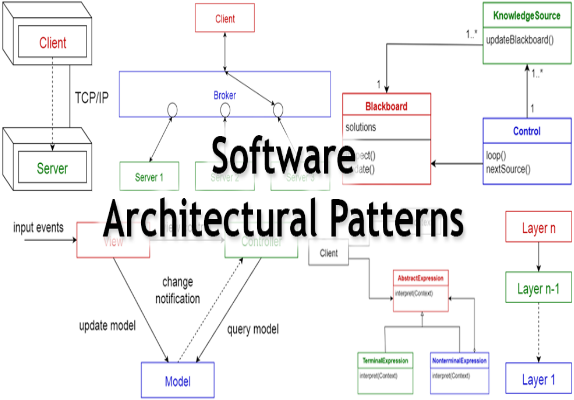
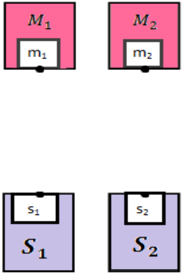


But how are they connected?


Software
Architectural Patterns
Client
TCP/IP
Server
Client
Broker
Server 1
KnowledgeSource
updateBlackboard()
1..*
1
Blackboard
solutions
Control
loop()
nextSource()
input events
View
Controller
change
notification
update model
query model
Model
Client
Layer n
Layer n-1
Layer 1

Master/Slave architecture

- Two types of components: masters and slaves.



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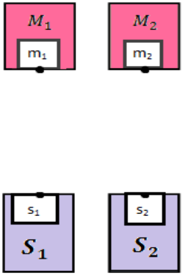

Properties of the architecture:

1. Masters interact only with slaves, and vice versa.
2. Each slave is connected to at most one master.

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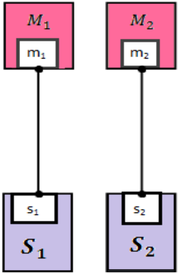

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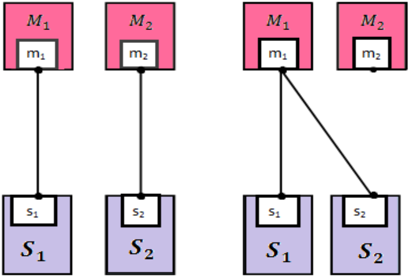

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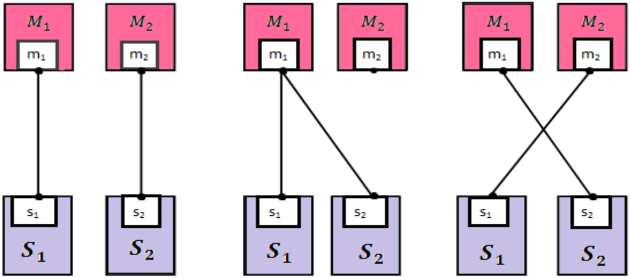

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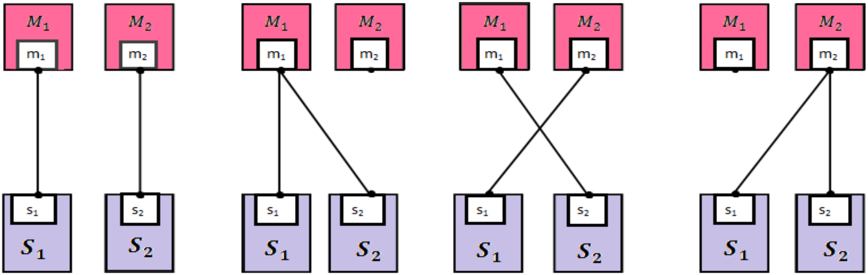

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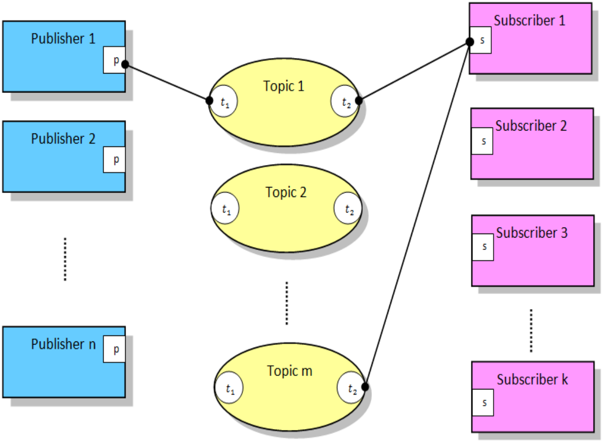
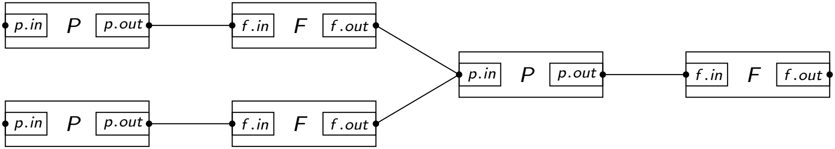
Publish/Subscribe architecture

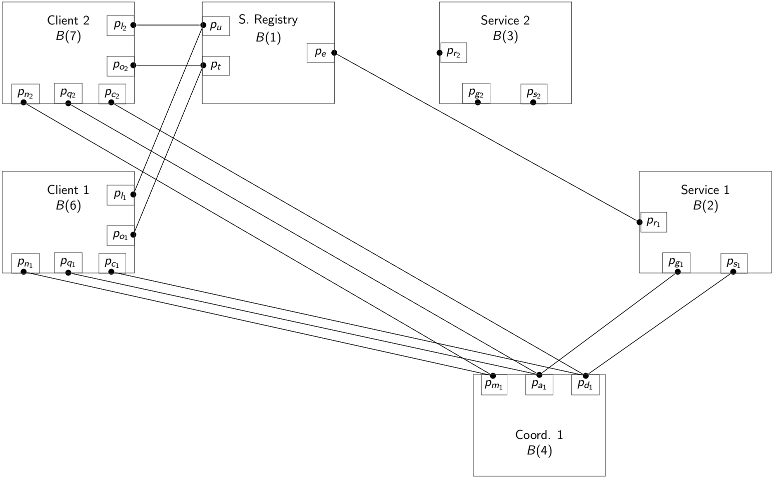


Pipes/Filters architecture

- The *p.out* port of any pipe can be connected to at most one filter port *f.in*.



Request/Response architecture



Question: How can we formally describe software architectures?

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Answer: Propositional Configuration Logic (PCL).

A. Mavridou, E. Baranov, S. Bliudze and J. Sifakis, **Configuration Logics: Modelling Architecture Styles**, in Journal of Logical and Algebraic Methods in Programming, vol. 86, num. 1, p. 2-29, 2016.

PCL is an extension of PIL

Definition

Considering a global set of ports P, an interaction is a non-empty set of ports $\alpha \subseteq P$ *such that* $\alpha \neq \emptyset$*. In other words an interaction* $\alpha \in I(P)$*, where* $I(P) = \mathcal{P}(P) \setminus \{\emptyset\}$ *and* $\mathcal{P}(P)$ *is the power set of P.*

The Propositional Interaction Logic is a Boolean logic used to characterize the interactions between components on a global set of ports P.

Syntax and semantics of *PIL*

<u>**Syntax:**</u> The PIL is defined by the grammar:

$$\phi ::= \textit{true} \mid p \mid \overline{\phi} \mid \phi \vee \phi$$

where $p \in P$.

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- $\alpha \models_i \overline{\phi}$ iff $\alpha \not\models_i \phi$.

PCL Syntax

The Propositional Configuration Logic is an extension of the *PIL* defined by the grammar:

$$f ::= true \mid \phi \mid \neg f \mid f \sqcup f \mid f + f$$

where:

- ϕ : *PIL* formula
- $\neg$: complementation operator
- $\sqcup$: union operator
- $+$: coalescing operator

PCL Semantics

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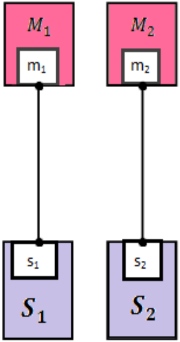
\gamma \models f_1 \sqcap f_2, & \text{if } \gamma \models f_1 \text{ and } \gamma \models f_2.
\end{array}
$$

Formally describing Master/Salve architecture

Consider the first Architecture scheme:

There are two interaction sets between its components:

- $\{m_1, s_1\}$
- $\{m_2, s_2\}$



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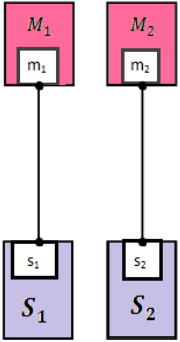

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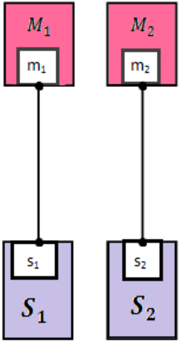

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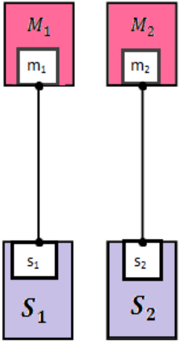
- $\{m_2, s_2\} \models_i \overbrace{m_2 \wedge s_2 \wedge \overline{m_1} \wedge \overline{s_1}}^{\phi_{22}}$



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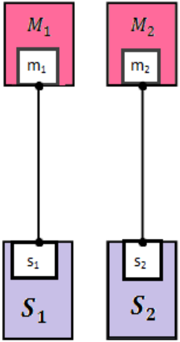
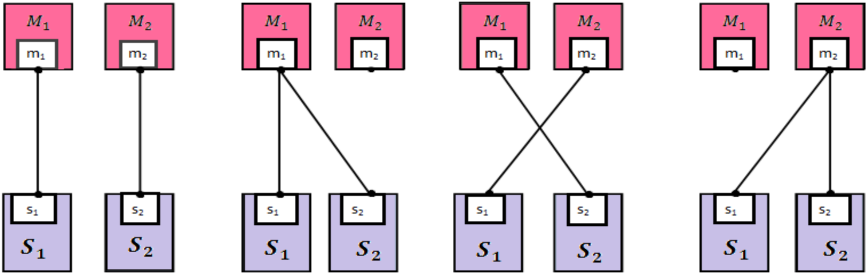
- $\{m_2, s_2\} \models_i \overbrace{m_2 \wedge s_2 \wedge \overline{m_1} \wedge \overline{s_1}}^{\phi_{22}}$

The configuration set that satisfies the architecture on the right is:

- $\{\{m_1, s_1\}, \{s_2, m_2\}\} \models \phi_{11} + \phi_{22}$



Given two slave components and two master components we get the following four architectures:



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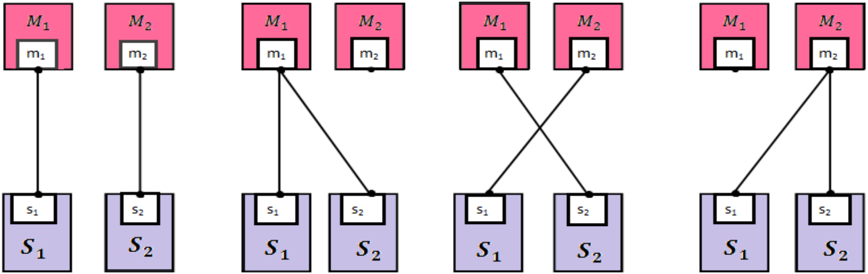
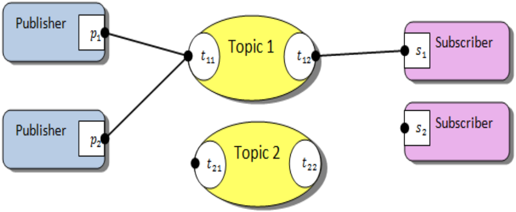


- The PCL formula f that describes the architecture is:

$$f = \bigsqcup_{i,i' \in \{1,2\}} (\phi_{1i} + \phi_{2i'})$$

where $\phi_{ji} = s_j \wedge m_i \wedge \overline{s_{j'}} \wedge \overline{m_{i'}}$ for $j, j' \in \{1, 2\}$ and $j \neq j'$.

Publish/Subscribe architecture



Let that there are two publishers, two topics and two subscribers.

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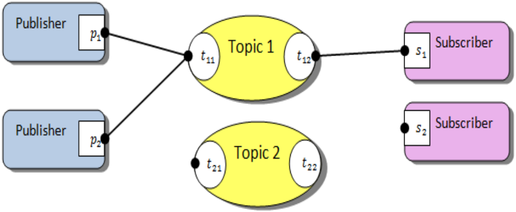

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- $f = \bigsqcup_{i,j\in\{1,2\}} \left(\left(s_i \wedge t_{j2} \wedge \bigwedge_{p\in P\setminus\{s_i,t_{j2}\}} \overline{p} \right) + \left(p_1 \wedge t_{j1} \wedge \bigwedge_{p\in P\setminus\{p_1,t_{j1}\}} \overline{p} \right) \sqcup \left(p_2 \wedge t_{j1} \wedge \bigwedge_{p\in P\setminus\{p_2,t_{j1}\}} \overline{p} \right) \right)$

Questions

- Given a PCL formula and a configuration set γ, is it always easy to show $\gamma \models f$ or $\gamma \not\models f$?
- Given two PCL formulas f_1, f_2 how can we decide whether $f_1 \equiv f_2$ or not ?

Definition

A full monomial is a monomial which involves all ports. A full monomial m is written as: $m = \bigwedge_{p \in P_+} p \wedge \bigwedge_{p \in P_-} \overline{p}$ *such that* $P = P_+ \cup P_-$ *and* $P_+ \cap P_- = \emptyset$.

For example, let $P = \{p, q, r, s, t\}$, which from the monomials below are full?

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For example, let $P = \{p, q, r, s, t\}$, which from the monomials below are full?

1. $pq\overline{r}st$ ✓
2. $pq\overline{r}t$ ✗
3. $prs\overline{q}t$ ✓

Full Normal Form

Definition

A PCL formula f is said to be in **Full Normal Form** *if it can be expressed in the following form:*

$$f \equiv \bigsqcup_{i \in I} \sum_{j \in J_i} m_{i,j}$$

where $m_{i,j}$ are **full** *monomials for every $i \in I$ and $j \in J_i$.*

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- The characteristic that makes full normal form very useful is that for every $i \in I$, there exists a unique configuration set γ_i such that $\gamma_i \models \sum_{j \in J_i} m_{i,j}$.

Theorem

Let P be a set of ports. Then, for every PCL formula f over P we can effectively construct, in doubly exponential time[a]*, an equivalent PCL formula f' in full normal form. The best run time for the construction of f' is exponential. Furthermore, f' is unique up to equivalence relation.*

[a] Complexity result proved in P. Paraponiari, G. Rahonis, Weighted propositional configuration logics: A specification language for architectures with quantitative features, *Inform. and Comput.* (accepted). Available at `https://arxiv.org/abs/1704.04969`.

Full normal form and Master/Slave architecture

The formula that describes the Master/Slave architecture is in full normal form:

$$f = (\phi_{11} + \phi_{22}) \sqcup (\phi_{11} + \phi_{21}) \sqcup (\phi_{12} + \phi_{22}) \sqcup (\phi_{12} + \phi_{21}) .$$

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The unique sets that satisfy f are:

- $\gamma_1 = \{\{s_1, m_1\}, \{s_2, m_2\}\}$
- $\gamma_2 = \{\{s_1, m_1\}, \{s_2, m_1\}\}$
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- Let a configuration set γ. Then $\gamma \models f$?
- $\gamma \models f$ iff there exists $i \in \{1, \ldots, n\}$ such that $\gamma = \gamma_i$.

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- $f_1$is equivalent to f_2 $(f_1 \equiv f_2)$ iff:

  1. $n_1 = n_2$ and
  2. for every $i \in \{1, \ldots, n_1\}$ there exists $j \in \{1, \ldots, n_1\}$ such that: $\gamma_i = \gamma'_j$.

Thank you