Logical directed description of software architectures with quantitative features

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(common work with G. Rahonis)

Seminar on Theoretical Computer Science and Discrete Mathematics
Aristotle University of Thessaloniki

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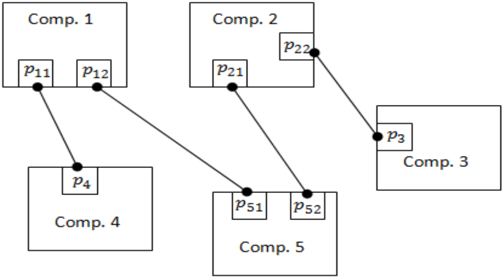
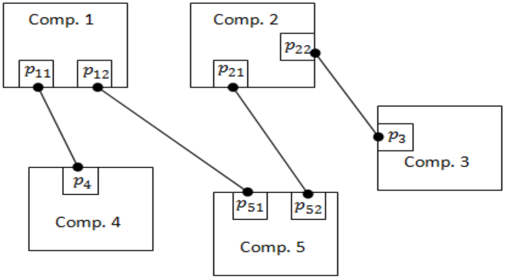
Software architectures



Software architectures

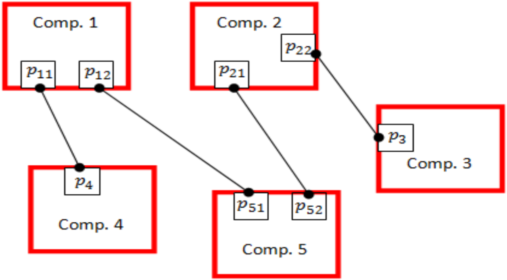
Consists of:



Software architectures

Consists of:

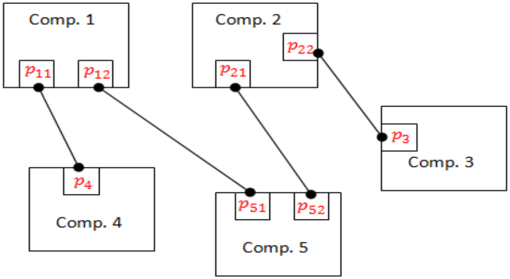
1. Components



Software architectures

Consists of:

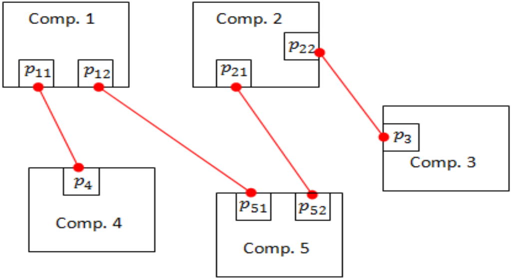
1. Components
2. Ports



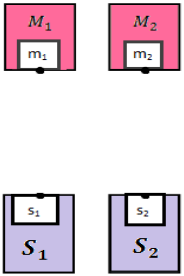
Software architectures

Consists of:

1. Components
2. Ports
3. Interactions



Master/Slave architecture



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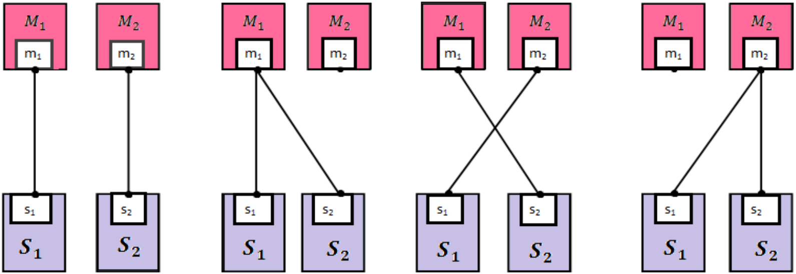
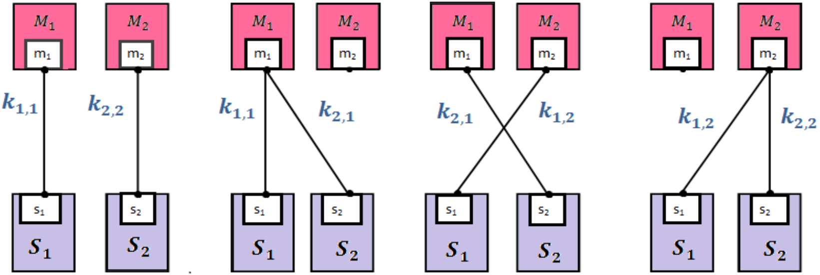
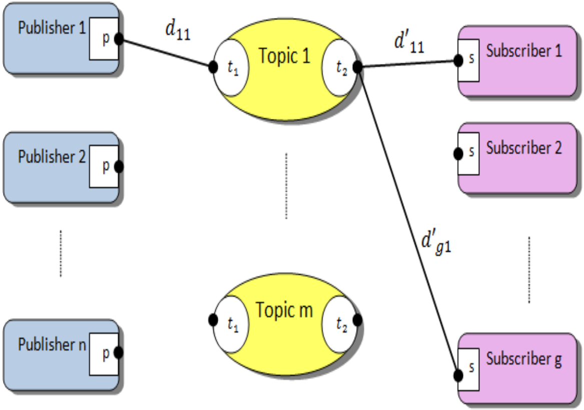


Master/Slave architecture with quantitative features



Publish/Subscribe architecture with quantitative features



- d_{ij} denotes the priority that the i publisher gives to the j topic and
- d'_{ij} denotes the priority that the i subscriber gives to the j topic.

Question

Propositional configuration logic (PCL) describes the quantitative features of a software architecture. What about the quantitative features? Can we compute the

1. the minimum cost or
2. the maximum probability or
3. the energy consumption

of a software architecture?

Tools we use

A semiring $(K, \oplus, \otimes, 0, 1)$ consists of a set K, two binary operations $\oplus$ and $\otimes$ and two constant elements 0 and 1 such that:

- $(K, \oplus, 0)$ is a commutative monoid,
- $(K, \otimes, 1)$ is a monoid,
- $\otimes$ distributes over $\oplus$,
- $0 \otimes k = k \otimes 0 = 0$ for every $k \in K$.

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♣ If the monoid $(K, \otimes, 1)$ is commutative, then the semiring is called **commutative**.

Syntax of the weighted propositional interaction logic (wPIL)

Let the semiring $(K, \oplus, \otimes, 0, 1)$. The syntax of formulas of the weighted PIL over P and K is given by the grammar:

$$\varphi ::= k \mid \phi \mid \varphi \oplus \varphi \mid \varphi \otimes \varphi$$

where $k \in K$ and ϕ denotes a PIL formula. We denote by:

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- $PIL(K, P)$ the set of all weighted PIL formulas φ over P and K.

Semantics of wPIL

Let $\varphi \in PIL(K, P)$ and $I(P) = \mathcal{P}(P) \setminus \{\emptyset\}$ where $\mathcal{P}(P)$ is the power set of P. The semantics of φ is a polynomial

$$\|\varphi\| : I(P) \to K.$$

For every $\alpha \in I(P)$ the value $\|\varphi\|(\alpha)$ is defined inductively as follows:

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Example

Let the semiring $(\mathbb{N}, +, \cdot, 0, 1)$ of natural numbers and the set $P = \{p, q, r\}$. We consider the *PIL* formulas:

- $\phi_1 = p \wedge q$
- $\phi_2 = p$

Let also $\alpha = \{p, q\}$, then the semantics of the formulas below are:

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- $\|\phi_1 \otimes \phi_2\|(\alpha) = \|\phi_1\|(\alpha) \otimes \|\phi_2\|(\alpha) = \|\phi_1\|(\alpha) \cdot \|\phi_2\|(\alpha) = 1 \cdot 1 = 1$

Syntax of the weighted Propositional configuration logic (wPCL)

The syntax of formulas of the **weighted PCL** over P and K is given by the grammar:

$$\zeta ::= k \mid f \mid \zeta \oplus \zeta \mid \zeta \otimes \zeta \mid \zeta \uplus \zeta$$

where:

- $k \in K$,
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- $\uplus$ denotes the coalescing operator among weighted PCL formulas.

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Also the set $PCL(K, P)$ will denote the set of all weighted PCL formulas over P and K.

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- $\|\zeta_1 \otimes \zeta_2\|(\gamma) = \|\zeta_1\|(\gamma) \otimes \|\zeta_2\|(\gamma)$,
- $\|\zeta_1 \uplus \zeta_2\|(\gamma) = \bigoplus_{\gamma = \gamma_1 \dot\cup \gamma_2} (\|\zeta_1\|(\gamma_1) \otimes \|\zeta_2\|(\gamma_2))$

  where $\dot\cup$ denotes the disjoint union of the sets γ_1 and γ_2.

Example on $\|\zeta_1 \uplus \zeta_2\|(\gamma)$

Consider the set of ports $P = \{p, q, r\}$ and the $PCL(K, P)$ formulas:

1. $\zeta_1 = pq \oplus r$,
2. $\zeta_2 = k \otimes p, \; k \in K$.

Let $\gamma = \{\{p, q\}, \{r\}\}$, then

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	γ_1	γ_2
1)	$\{\{p, q\}\}$	$\{\{r\}\}$
2)	$\{\{r\}\}$	$\{\{p, q\}\}$

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$\|\zeta_1\|(\gamma_1)$	$\|\zeta_2\|(\gamma_2)$	$\otimes$
1	0	0
1	k	k

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Closure operator

Definition

The **closure** $\sim \zeta$ *of every weighted PCL formula* $\zeta \in PCL(K, P)$ *is:*

$$\sim \zeta := \zeta \oplus (\zeta \uplus 1).$$

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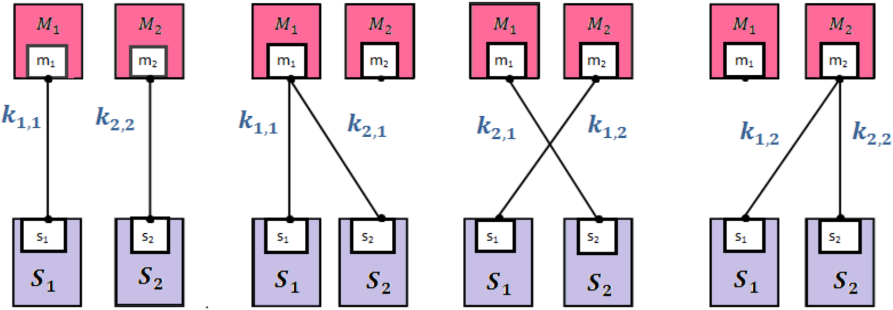
Let $\gamma \in C(P)$, then the semantics of $\sim \zeta$ is:

$$\|{\sim \zeta}\|(\gamma) = \bigoplus_{\gamma' \subseteq \gamma} \|\zeta\|(\gamma')$$

Application on weighted Master/Slave architecture

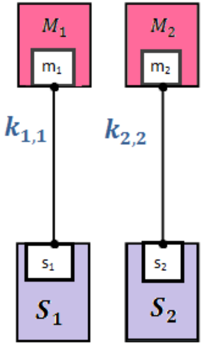
We consider that each interaction has some kind of “cost”.



Consider the first Architecture scheme:

There are two interactions between its components:

- $\{m_1, s_1\}$:
- $\{m_2, s_2\}$:



Consider the first Architecture scheme:

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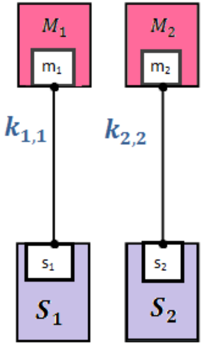
- $\{m_1, s_1\}$: $(s_1 \wedge m_1 \wedge \overline{s_2} \wedge \overline{m_2})$
- $\{m_2, s_2\}$:



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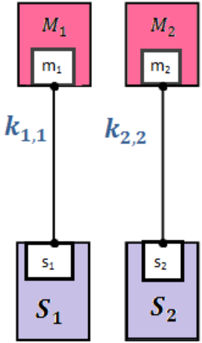
- $\{m_1, s_1\}$: $k_{11} \otimes (s_1 \wedge m_1 \wedge \overline{s_2} \wedge \overline{m_2})$
- $\{m_2, s_2\}$:



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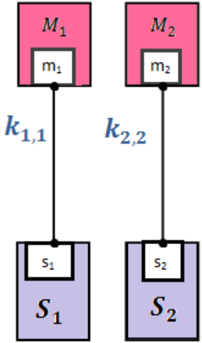
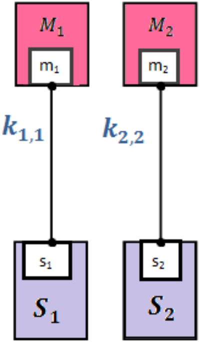
- $\{m_1, s_1\}$: $k_{11} \otimes (s_1 \wedge m_1 \wedge \overline{s_2} \wedge \overline{m_2})$
- $\{m_2, s_2\}$: $k_{22} \otimes (s_2 \wedge m_2 \wedge \overline{s_1} \wedge \overline{m_1})$



Consider the first Architecture scheme:

There are two interactions between its components:

- $\{m_1, s_1\}$: $\overbrace{k_{11} \otimes (s_1 \wedge m_1 \wedge \overline{s_2} \wedge \overline{m_2})}^{\varphi_{1,1}}$
- $\{m_2, s_2\}$: $\overbrace{k_{22} \otimes (s_2 \wedge m_2 \wedge \overline{s_1} \wedge \overline{m_1})}^{\varphi_{2,2}}$



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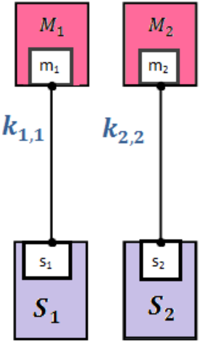

The wPCL formula that formalizes the architecture on the right is:

$$\varphi_{1,1} \uplus \varphi_{2,2}$$

The weighted *PCL* formula that formalizes the four architecture schemes is:

$$\zeta = (\varphi_{1,1} \oplus \varphi_{1,2}) \uplus (\varphi_{2,1} \oplus \varphi_{2,2})$$

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$$\zeta = (\varphi_{1,1} \oplus \varphi_{1,2}) \uplus (\varphi_{2,1} \oplus \varphi_{2,2})$$

We consider the wPCL formula

$$\sim \zeta$$

and the configuration set $\gamma \in C(P)$. Then, depending on the semiring, the value $\|\sim \zeta\| (\gamma)$ changes meaning.

1. $K = \mathbb{R}_{\min} = (\mathbb{R}_+ \cup \{\infty\}, \min, +, \infty, 0)$, then

$$\| \sim \zeta \| (\gamma)$$

returns the minimum "cost" for the implementation of the Master/Slave architecture style with input γ.

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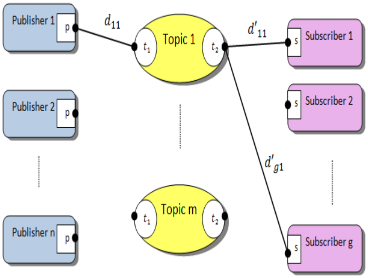
returns the minimum "cost" for the implementation of the Master/Slave architecture style with input γ.

2. $K = ([0,1], \max, \cdot, 0, 1)$ the value

$$\|\sim \zeta\|(\gamma)$$

represents the configuration to be implemented with the maximum probability for the input γ.

Publish/Subscribe architecture with quantitative features



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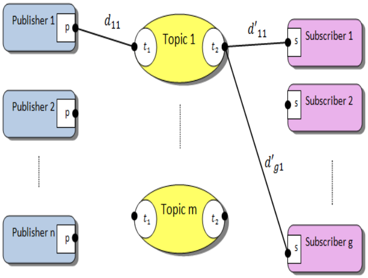

For every subscriber S_i we constructed a wPCL formula ζ_{s_i} which describes the weighted interactions with the rest components. If we consider a set $\gamma \in C(P)$ and the semiring

$$K = (\mathbb{R}_+ \cup \{-\infty\}, \max, +, -\infty, 0),$$

the value

$$\|\zeta_{s_i}\|(\gamma)$$

represents the maximum priority with which the subscriber will receive a message.

Application on the travelling salesman problem

The **travelling salesman problem** is a problem where given a list of cities and the distances between each pair of cities, we want to find the shortest possible route that visits each city exactly once and returns to the origin city.

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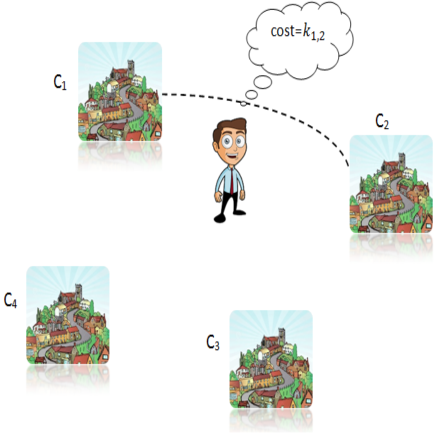
The **travelling salesman problem** is a problem where given a list of cities and the distances between each pair of cities, we want to find the shortest possible route that visits each city exactly once and returns to the origin city.

In this application we will examine the problem for only 4 cities and we will show the weighted formula that formalises all the possible routes. Also the weight in our formula can either represent **distance**, **cost** of the means of transport, **gas** in our car e.t.c.



Let the *PIL* formulas:

$$\phi_{i,j} = c_i \wedge c_j \wedge \bigwedge_{n \neq i,j} \overline{c_n}$$

for every $i, j \in \{1, 2, 3, 4\}$, that characterise the interactions between pair of cities. Consider that each interaction has some kind of 'cost' and let the weighted *PIL* formlulas:

$$\varphi_{i,j} = k_{i,j} \otimes \phi_{i,j}$$

for every $i, j \in \{1, 2, 3, 4\}$ and $i \neq j$.

The $PCL(K,P)$ formula that formalises all the possible routes is:

$$\zeta = (\varphi_{1,2} \uplus \varphi_{2,3} \uplus \varphi_{3,4} \uplus \varphi_{1,4}) \oplus (\varphi_{1,2} \uplus \varphi_{2,4} \uplus \varphi_{3,4} \uplus \varphi_{1,3}) \oplus$$
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then the semantics $\|\sim \zeta\| (\gamma)$, computes all the possible semantics of the formula ζ for every possible $\gamma' \subseteq \gamma$ and then computes the minimum one. That way we can compute the least cost of all the possible routes.

Questions

- Given a wPCL formula ζ and a configuration set γ, is it always easy to find the value $\|\zeta\|(\gamma)$?
- Given two wPCL formulas ζ_1, ζ_2 is it decidable whether $\zeta_1 \equiv \zeta_2$ or not ?

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Definition

Two weighted PCL formulas ζ_1, ζ_2 *are called* **equivalent**, *and we write:*

$$\zeta_1 \equiv \zeta_2 \textit{ whenever } \|\zeta_1\|(\gamma) = \|\zeta_2\|(\gamma)$$

for every $\gamma \in C(P)$.

Full normal form for weighted *PCL* formulas

Definition

A weighted PCL formula $\zeta \in PCL(K, P)$ is said to be in *full normal form* if there are finite index sets I and J_i for every $i \in I$, $k_i \in K$ for every $i \in I$, and full monomials $m_{i,j}$ for every $i \in I$ and $j \in J_i$ such that

$$\zeta = \bigoplus_{i \in I} \left(k_i \otimes \sum_{j \in J_i} m_{i,j} \right).$$

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- The characteristic that makes full normal form very useful is that for every $i \in I$, there exists a unique configuration set γ_i such that

$$\|\zeta\| (\gamma_i) = k_i$$

and $\|\zeta\| (\gamma) = 0$ for every $\gamma \in C(P)$ such that $\gamma \neq \gamma_i$ for every $i \in I$.

Theorem

Let K be a commutative semiring and P a set of ports. Then for every weighted PCL formula $\zeta \in PCL(K, P)$ we can effectively construct an equivalent weighted PCL formula $\zeta' \in PCL(K, P)$ in full normal form which is **unique** *up to the equivalence relation. The worst case run time for the construction algorithm is doubly exponential and the best case is exponential.*

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- ζ_1 is equivalent to ζ_2 iff:

  1. $n_1 = n_2$ and
  2. for every $i \in \{1, \ldots, n_1\}$ there exists $j \in \{1, \ldots, n_1\}$ such that: $\gamma_i = \gamma'_j$ and $k_i = k'_j$.

Thank you!